REPORT DOCUMENTATION PAGE

AD-A256 396

1. Agency Use Only (*Leave blank*).	2. Report Date. July 1991	3. Re[port Type and Dates Covered.] Proceedings

4. Title and Subtitle.
Hamilton's Principle and the Equations of Motion of An Elastic Shell With and Without Fluid Loading[1]

5. Funding Numbers.
Program Element No. 61153N
Project No. 03202
Task No. 010
Accession No. DN255011

2

6. Author(s).
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7. Performing Organization Name(s) and Address(es).
Naval Oceanographic and Atmospheric Research Laboratory
Ocean Acoustics and Technology Directorate
Stennis Space Center, MS 39529-5004

8. Performing Organization Report Number.
PR 91:068:221

9. Sponsoring/Monitoring Agency Name(s) and Address(es).
Naval Oceanographic and Atmospheric Research Laboratory
Basic Research Management Office
Stennis Space Center, MS 39529-5004

10. Sponsoring/Monitoring Agency Report Number.
PR 91:068:221

11. Supplementary Notes.
IMACS '91

12a. Distribution/Availability Statement.
Approved for public release; distribution is unlimited.

12b. Distribution Code.

13. Abstract (*Maximum 200 words*).

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14. Subject Terms.
Acoustic scattering, shallow water, waveguide propagation

15. Number of Pages. 2

16. Price Code.

17. Security Classification of Report.	18. Security Classification of This Page.	19. Security Classification of Abstract.	20. Limitation of Abstract.
Unclassified	Unclassified	Unclassified	SAR

REPRINTED FROM:

Computational and Applied Mathematics, II

Differential Equations

Selected and revised papers from the
IMACS 13th World Congress,
Dublin, Ireland, July 1991

edited by

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School of Mathematics
Georgia Institute of Technology
Atlanta, GA, U.S.A.

P.J. VAN DER HOUWEN
Centrum voor Wiskunde en Informatica
Stichting Mathematisch Centrum
Amsterdam, The Netherlands

1992

NORTH-HOLLAND
AMSTERDAM • LONDON • NEW YORK • TOKYO

Computational and Applied Mathematics, II
W.F. Ames and P.J. van der Houwen (Editors)
Elsevier Science Publishers B.V. (North-Holland)

Hamilton's principle and the equations of motion of an elastic shell with and without fluid loading

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Abstract

It has proven quite difficult to employ exact elastodynamic theory to describe the behavior of elastic vibrations on arbitrary bounded shells. In addition, exact theories preclude direct interpretation of particular features observed due to the excitation of elastic shell surfaces. A rather interesting approach to describe surface vibrations may be obtained by constructing a Hamiltonian in some approximate form that assumes some correlation of motion of the outer and inner shell surface. The class of theories that allow for this approach are referred to in applied mechanics as shell theories. The interesting feature of this Hamiltonian approach is that one can add various physical mechanisms to the Hamiltonian such as extensional motion, rotary inertia, fluid loading, etc., and thereby study the individual contributions to resonance patterns while adding physical insight to the fundamental processes that occur on shell surfaces. We develop shell theories in this manner and examine various contributions via Hamilton's principle. We believe that fluid loading has by and large not been treated adequately in the past, and we place particular emphasis on the treatment of that contribution to this work.

1. INTRODUCTION

Sound scattering from submerged elastic shells is of interest to a broad community of scientists and engineers. For example in the area of nondestructive testing ultrasonic scattering is of considerable interest to the structural engineering community, while in the area of remote sensing or object identification scattering from objects proves invaluable. The acoustical scattering from elastic objects or more simply the generation of resonances from bounded elastic shells involves the use of acoustic and elastodynamic theory. Exact solutions for scattering from elastic targets exist for target shapes for which the elastodynamic equation is separable. For three dimensional targets separability is only possible for spherical and rectangular targets. For more complicated targets numerical techniques must be used. For axisymmetric smooth targets the extended boundary method of Waterman has proven useful though limited and an approximate theory would be desirable for more complicated targets. The purpose of this work is to develop an approximate theory based on shell theory that would be of use for general shapes that could also include structural loading. Shell theories afford a very powerful methodology to "build in" structural features usually via a variational principle in which some Largrangian or Hamiltonian is constructed by introducing physical features. The simplest theories which always assume correlated motion of the inner and outer surfaces of the target include kinematic features and potential energy terms based on the generalized Hooke's law which leads to the lowest order symmetric mode. One can add rotational inertia to the Hamiltonian which allows an antisymmetric mode. However the antisymmatric mode with just this added feature does not have the correct asymptotic behavior. Timoshenko noted that by

including shear distortion the antisymmetric mode obeyed the correct asymptotic features. One goes on from there to include higher order corrections. Further, proper fluid loading -- which from our experience is no easy task -- must be introduced correctly. The proper inclusion of fluid loading introduces a fluid borne wave: the pseudo-Stoneley wave. Clearly thin shell theories are rather geometrical and must be constructed for each shape due to their dependence on shape dependent dynamic factors. Non-spherical three dimensional objects present a problem which we seek to address in the future. In this work we wish to develop and test a suitable shell theory for spherical shells that can be readily generalized to spheroids and finite cylinders. In the following section we outline the derivation of a shell theory with many of the featured described. The theories, with various levels of sophistication, are then used to compare with the exact method. The results are then discussed and future work is described.

2. DERIVATION OF EQUATIONS OF MOTION

In spherical shells membrane stresses (proportional to β) predominate over flexural stresses (proportional to β^2) where

$$\beta = \frac{1}{\sqrt{12}}\frac{h}{a}. \tag{1}$$

We differ from the standard derivation for the sphere [2] by retaining all terms of order β^2 in both the kinetic and potential energy parts of the Lagrangian and by considering the resonance frequencies for the fluid loaded case to be complex. We note that this level of approximation will allow us to include the effects of rotary inertia in our shell theory, as well as damping by fluid loading. The parameter β itself is proportional to the radius of gyration of a differential element of the shell and arises from integration through the thickness of the shell in a radial direction. We will use an implicit harmonic time variation of the form $\exp(-i\omega t)$. We begin our derivation by considering a u,v,w axis system on the middle surface of a spherical shell of radius a (measured to mid-shell) with thickness h, where u increases meridionally southward, v increases latitudinally eastward, and w increases radially outward.

2.1. Lagrangian Variational Analysis

Our Lagrangian, L, is

$$L = T - V + W, \tag{2}$$

where T is the kinetic energy, V is the potential energy, and W is the work due to the pressure at the surface. The kinetic energy is given by

$$T = \frac{1}{2}\rho_s \int_0^{2\pi}\int_0^{\pi}\int_{-h/2}^{h/2} (\dot{u}_s^2 + \dot{w}_s^2)(a+x)^2 \sin\theta \, dx\, d\theta\, d\phi, \tag{3}$$

where the surface displacements are taken to be linear:

$$\dot{u}_s = (1+\frac{x}{a})\dot{u} - \frac{x}{a}\frac{\partial \dot{w}}{\partial \theta}, \tag{4}$$

and

$$\dot{w}_s = \dot{w}. \tag{5}$$

The motion of the spherical shell is axisymmetric since the sound field is torsionless. Thus there is no motion in the v-direction. Substitution of Eqs. (4) and (5) into Eq. (3) yields, after integration over x and ϕ,

$$T = \pi\rho_s \int_0^\pi \sin\theta[(\frac{h^5}{80a^2} + \frac{h^3}{2} + ha^2)\dot{u}^2 - 2(\frac{h^5}{80a^2} + \frac{h^3}{4})\dot{u}\frac{\partial\dot{w}}{\partial\theta}$$

$$+(\frac{h^5}{80a^2} + \frac{h^3}{12})(\frac{\partial\dot{w}}{\partial\theta})^2 + (\frac{h^3}{12} + ha^2)\dot{w}^2]d\theta, \tag{6}$$

or, in terms of β,

$$T = \pi\rho_s ha^2 \int_0^\pi [(1.8\beta^4 + 6\beta^2 + 1)\dot{u}^2 - (3.6\beta^4 + 6\beta^2)\dot{u}\frac{\partial\dot{w}}{\partial\theta}$$

$$+(1.8\beta^4 + \beta^2)(\frac{\partial\dot{w}}{\partial\theta})^2 + (\beta^2 + 1)\dot{w}^2]\sin\theta d\theta, \tag{7}$$

where the first and last terms in square brackets in Eq. (7) are associated with linear translational kinetic energies and the middle two terms are associated with rotational kinetic energies of an element of the shell.

The potential energy of the shell is

$$V = \frac{1}{2}\int_0^\pi \int_0^{2\pi} \int_{-h/2}^{h/2} (\sigma_{\theta\theta}\varepsilon_{\theta\theta} + \sigma_{\phi\phi}\varepsilon_{\phi\phi})(x+a)^2 \sin\theta dx d\theta d\phi, \tag{8}$$

where the nonvanishing components of the strain are

$$\varepsilon_{\theta\theta} = \frac{1}{a}\left(\frac{\partial u}{\partial\theta} + w\right) + \frac{x}{a^2}\left(\frac{\partial u}{\partial\theta} - \frac{\partial^2 w}{\partial\theta^2}\right), \tag{9}$$

and

$$\varepsilon_{\phi\phi} = \frac{1}{a}(\cot\theta u + w) + \frac{x}{a^2}\cot\theta\left(u - \frac{\partial w}{\partial\theta}\right), \tag{10}$$

and where the nonzero stress components are

$$\sigma_{\theta\theta} = \frac{E}{1-\nu^2}(\varepsilon_{\theta\theta} + \nu\varepsilon_{\phi\phi}), \tag{11}$$

and

$$\sigma_{\phi\phi} = \frac{E}{1-\nu^2}(\varepsilon_{\phi\phi} + \nu\varepsilon_{\theta\theta}), \tag{12}$$

where E is Young's modulus. By substitution the potential energy becomes

$$V = \frac{1}{2}\int_0^{\pi}\int_0^{2\pi}\int_{-h/2}^{h/2}\left[\frac{E}{1-\nu^2}\frac{1}{(x+a)^2}\left([(1+\frac{x}{a})\frac{\partial u}{\partial\theta} - \frac{x}{a}\frac{\partial^2 w}{\partial\theta^2} + w^2] + \{\cot\theta[(1+\frac{x}{a})u - \frac{x}{a}\frac{\partial w}{\partial\theta}] + w\}^2\right.\right.$$

$$\left.\left. +2\nu\{\cot\theta[(1+\frac{x}{a})u - \frac{x}{a}\frac{\partial w}{\partial\theta}] + w\}[(1+\frac{x}{a})\frac{\partial u}{\partial\theta} - \frac{x}{a}\frac{\partial^2 w}{\partial\theta^2} + w]\right)\right](x+a)^2\sin\theta dx d\theta, \tag{13}$$

which after integration is

$$V = \frac{\pi E h}{1-\nu^2}\int_0^{\pi}\{(w+\frac{\partial u}{\partial\theta})^2 + (w+u\cot\theta)^2 + 2\nu(w+\frac{\partial u}{\partial\theta})(w+u\cot\theta)$$

$$+\beta^2[(\frac{\partial u}{\partial\theta} - \frac{\partial^2 w}{\partial\theta^2})^2 \cot^2\theta(u-\frac{\partial w}{\partial\theta})^2 + 2\nu\cot\theta(u-\frac{\partial w}{\partial\theta})(\frac{\partial u}{\partial\theta} - \frac{\partial^2 w}{\partial\theta^2})]\}\sin\theta d\theta. \tag{14}$$

Terms in the potential energy proportional to β^2 are due to bending stresses.

And finally, the work done by the pressure of the surrounding fluid on the spherical shell is given by

$$W = 2\pi a^2\int_0^{\pi} p_a w \sin\theta d\theta, \tag{15}$$

where p_a is the pressure at the surface.

2.2. The Lagrangian density and its equations of motion

Integration along the polar angle θ is intrinsic to the problem, therefore we must turn to a *Lagrangian density* formulation to solve for the equations of motion. Our Lagrangian density is just

$$\begin{aligned} \mathcal{L} = {} & \pi\rho_s h a^2[(1+6\beta^2+1.8\beta^4)\dot{u}^2 - (6\beta^2+3.6\beta^4)\dot{u}\frac{\partial\dot{w}}{\partial\theta} + (\beta^2+1.8\beta^4)(\frac{\partial\dot{w}}{\partial\theta})^2 \\ & +(1+\beta^2)\dot{w}^2]\sin\theta - \frac{\pi E h}{1-\nu^2}\{(w+\frac{\partial u}{\partial\theta})^2 + (w+u\cot\theta)^2 + 2\nu(w+\frac{\partial u}{\partial\theta})(w+u\cot\theta) \\ & +\beta^2[(\frac{\partial u}{\partial\theta} - \frac{\partial^2 w}{\partial\theta^2})^2 + \cot^2\theta(u-\frac{\partial w}{\partial\theta})^2 + 2\nu\cot\theta(u-\frac{\partial w}{\partial\theta})(\frac{\partial u}{\partial\theta} - \frac{\partial^2 w}{\partial\theta^2})]\}\sin\theta \\ & +2\pi a^2 p_a w\sin\theta, \end{aligned} \tag{16}$$

with corresponding differential equations

$$0 = \frac{\partial \mathcal{L}}{\partial u} - \frac{d}{d\theta}\frac{\partial \mathcal{L}}{\partial u_\theta} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial u_t}, \tag{17}$$

and

$$0 = \frac{\partial \mathcal{L}}{\partial w} - \frac{d}{d\theta}\frac{\partial \mathcal{L}}{\partial w_\theta} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial w_t} + \frac{d^2}{d\theta dt}\frac{\partial \mathcal{L}}{\partial w_{\theta t}} + \frac{d^2}{d\theta^2}\frac{\partial \mathcal{L}}{\partial w_{\theta\theta}}, \tag{18}$$

where subscripts denote differentiation of the variable with respect to the subscript.

By substitution of Eqs. (17) and (18) into (16) we obtain

$$\begin{aligned}0 &= (1+\beta^2)\left[\frac{\partial^2 u}{\partial \theta^2} + \cot\theta\frac{\partial u}{\partial\theta} - (\nu + \cot^2\theta)u\right] - \beta^2\frac{\partial^3 w}{\partial\theta^3} - \beta^2\cot\theta\frac{\partial^2 w}{\partial\theta^2}\\ &+[(1+\nu)+\beta^2(\nu+\cot^2\theta)]\frac{\partial w}{\partial\theta} - \frac{a^2}{c_p^2}[(1.8\beta^4+6\beta^2+1)\frac{\partial^2 u}{\partial t^2} - (1.8\beta^4+3\beta^2)\frac{\partial^3 w}{\partial\theta\partial t^2}],\end{aligned} \tag{19}$$

and

$$\begin{aligned}-p_a\frac{(1-\nu^2)a^2}{Eh} &= \beta^2\frac{\partial^3 u}{\partial\theta^3} + 2\beta^2\cot\theta\frac{\partial^2 u}{\partial\theta^2} - [(1+\nu)(1+\beta^2)+\beta^2\cot^2\theta)]\frac{\partial u}{\partial\theta}\\ &+\cot\theta[(2-\nu+\cot^2\theta)\beta^2-(1+\nu)]u - \beta^2\frac{\partial^4 w}{\partial\theta^4} - 2\beta^2\cot\theta\frac{\partial^3 w}{\partial\theta^3}\\ &+\beta^2(1+\nu+\cot^2\theta)\frac{\partial^2 w}{\partial\theta^2} - \beta^2\cot\theta(2-\nu+\cot^2\theta)\frac{\partial w}{\partial\theta} - 2(1+\nu)w\\ &+\frac{a^2}{c_p^2}[-(1.8\beta^4+3\beta^2)\frac{\partial^3 u}{\partial\theta\partial t^2} - (1.8\beta^4+3\beta^2)\cot\theta\frac{\partial^2 u}{\partial t^2}\\ &+(1.8\beta^4+\beta^2)\frac{\partial^4 w}{\partial\theta^2\partial t^2} + (1.8\beta^4+\beta^2)\frac{\partial^3 w}{\partial\theta\partial t^2}\cot\theta - (\beta^2+1)\frac{\partial^2 w}{\partial t^2}.\end{aligned} \tag{20}$$

These differential equations of motion (19) and (20) have solutions of the form

$$u(\eta) = \sum_{n=0}^{\infty} U_n(1-\eta^2)^{1/2}\frac{dP_n}{d\eta}, \tag{21}$$

and

$$w(\eta) = \sum_{n=0}^{\infty} W_n P_n(\eta), \tag{22}$$

where $\eta = \cos\theta$ and $P_n(\eta)$ are the Legendre polynomials of the first kind of order n. When the differential equations of motion (19) and (20) are expanded in terms of Eqs. (21) and (22), we obtain a set of linear equations in terms of U_n and W_n, whose determinant must vanish. We shall consider two cases: with and without fluid loading.

2.3. Vacuum case

The simpler case is that when the spherical shell is surrounded by a vacuum such that there is no damping. In this case, the pressure at the surface vanishes: $p_a = 0$. The set of linear equations the expansion coefficients must satisfy are

$$0 = [\Omega^2(1+6\beta^2+1.8\beta^4)-(1+\beta^2)\kappa]U_n + [\Omega^2(3\beta^2+1.8\beta^4)-\beta^2\kappa-(1+\nu)]W_n, \tag{23}$$

and

$$0 = -\lambda_n[(\kappa-3)\beta^2-1.8\beta^4+1+\nu]U_n + [\Omega^2(1+2\beta^2+1.8\beta^4)-2(1+\nu)-\beta^2\kappa\lambda_n]W_n, \tag{24}$$

where $\Omega = \omega a / c_p$, $\kappa = \nu + \lambda_n - 1$, and $\lambda_n = n(n+1)$. In order for Eqs. (23) and (24) to be satisfied simultaneously with a non-trivial solution the determinant of the system must vanish:

$$\begin{aligned}0 &= \Omega^4(1+6\beta^2+1.8\beta^4)(1+2\beta^2+1.8\beta^4)\\ &+\Omega^2\{(3\beta^2+1.8\beta^4)\lambda_n[(\kappa-3)\beta^2-1.8\beta^4+1+\nu]\\ &-[2(1+\nu)+\beta^2\kappa\lambda_n](1+6\beta^2+1.8\beta^4)-(1+\beta^2)\kappa(1+2\beta^2+1.8\beta^4)\}\\ &+(1+\beta^2)\kappa[2(1+\nu)+\beta^2\kappa\lambda_n]-\lambda_n[(\kappa-3)\beta^2-1.8\beta^4+1+\nu](\beta^2\kappa+1+\nu).\end{aligned} \tag{25}$$

Since there are no damping terms, the shell vibrates theoretically forever. Thus, the normalized frequency Ω can be taken to be real. Equation (25) is quadratic in Ω^2, thus we expect two real roots to (25) and thus two modes for the motion of the shell. They are the symmetric and antisymmetric modes.

2.4. Fluid loaded case

For the fluid loaded case, we must consider a modal expansion of the surface pressure in terms of the specific acoustic impedance z_n. In its most general form this is

$$p(a,\theta,\phi) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} z_n \dot{W}_{mn} P_n^m(\cos\theta)\cos m\phi, \tag{26}$$

where

$$z_n = i\rho c \frac{h_n(ka)}{h_n'(ka)}. \tag{27}$$

The specific acoustic impedance z_n can be split into real and imaginary parts:

$$z_n = r_n - i\omega m_n, \tag{28}$$

where

$$r_n = \rho c \,\mathrm{Re}\left\{\frac{ih_n(ka)}{h_n'(ka)}\right\}, \tag{29}$$

and

$$m_n = -\frac{\rho c}{\omega}\mathrm{Im}\left\{\frac{ih_n(ka)}{h_n'(ka)}\right\}. \tag{30}$$

For the case we are considering of axisymmetric motion, the surface pressure is given by

$$p_a(\theta) = -\sum_{n=0}^{\infty} z_n \dot{W}_n P_n(\cos\theta), \tag{31}$$

or

$$p_a(\theta) = -\sum_{n=0}^{\infty}(-i\omega W_n r_n - \omega^2 W_n m_n)P_n(\cos\theta). \tag{32}$$

Use of Eq. (32) in our set of differential equations of motion (19) and (20) yields the following set of linear equations for the expansion coefficients in the case of a fluid loaded spherical shell:

$$0 = [\Omega^2(1+6\beta^2+1.8\beta^4)-(1+\beta^2)\kappa]U_n + [\Omega^2(3\beta^2+1.8\beta^4)-\beta^2\kappa-(1+\nu)]W_n, \tag{33}$$

and

$$\begin{aligned}0 = &-\lambda_n[(\kappa-3)\beta^2-1.8\beta^4+1+\nu]U_n\\ &+[\Omega^2(1+\alpha+2\beta^2+1.8\beta^4)-2(1+\nu)+\Omega i\gamma-\beta^2\kappa\lambda_n]W_n,\end{aligned} \tag{34}$$

where

$$\alpha = \frac{m_n}{\rho_s h}, \tag{35}$$

and

$$\gamma = \frac{a}{h}\frac{r_n}{\rho_s c_p}. \tag{36}$$

Again the determinant of Eqs. (33) and (34) must vanish. However, in this instance the value of Ω must be taken to be complex; the resonances have a width that depends on the damping. The result of setting this determinant to zero is

$$\begin{aligned}
0 &= \Omega^4(1+6\beta^2+1.8\beta^4)(1+\alpha+2\beta^2+1.8\beta^4) \\
&+\Omega^3 i\gamma(1+6\beta^2+1.8\beta^4) \\
&+\Omega^2\{(3\beta^2+1.8\beta^4)\lambda_n[(\kappa-3)\beta^2-1.8\beta^4+1+\nu] \\
&-[2(1+\nu)+\beta^2\kappa\lambda_n](1+6\beta^2+1.8\beta^4)-(1+\beta^2)\kappa(1+\alpha+2\beta^2+1.8\beta^4)\} \\
&+\Omega[-i\gamma(1+\beta^2)\kappa] \\
&+(1+\beta^2)\kappa[2(1+\nu)+\beta^2\kappa\lambda_n]-\lambda_n[(\kappa-3)\beta^2-1.8\beta^4+1+\nu](\beta^2\kappa+1+\nu).
\end{aligned} \tag{37}$$

Equation (37) has four complex roots. From work with an exact modal solution to the problem, we expect two roots to be associated with the symmetric and antisymmetric modes of the shell. We expect the other two roots to be associated with a water-borne pseudo-Stoneley wave.

3. CONCLUSIONS

The next step is to plot the roots of Eqs. (25) and (37) to compare the resonances predicted by these models with those given by exact modal expansion solutions. By suppressing α and γ, the model associated with Eq. (37) reverts to the vacuum case model associated with Eq. (25). Similarly suppression of factors of β in Eq. (25) will result in a reversion to a previously derived solution [1]. We may then rank the three different models according to their degree of physicality and compare their results for various relative shell thicknesses against each other and against the exact results of the modal expansion model. We may also consider the limitations of each of the models including the exact solution, as well as those of shell models in general.

These models, fluid loaded, vacuo case, and membrane, are successively less physically sophisticated and give successively less good comparison with exact (modal expansion) results. Starting with the least sophisticated model, we see in Fig. 1 thick spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by the membrane model. Here and in the succeed'ng figures thick means $h/a = 0.1$; thin means $h/a = 0.01$. The shell material is a generic steel with density $\rho_s = 7.7$ times that of water, shear velocity $v_s = 3.24$ km/s, and longitudinal velocity $v_l = 5.95$ km/s. The surrounding fluid is taken to be water with density $\rho = 1000$ kg/m^3 and sound velocity $c_w = 1.4825$ km/s. The symmetric mode shows a good comparison between exact and shell theory predictions, but the antisymmetric shell theory results for this approximation compare poorly with the exact flexural results. Note that some symmetric mode resonances were not found by our exact theory algorithm. In Fig. 2 we see thin spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by the membrane model. Again there is good comparison between dilatational (symmetric) mode resonances calculated by the two methods, except for the first couple of resonances. The exact flexural resonances are highly damped; the few resonances seen are water-borne pseudo-Stonely resonances. And again the shell theory flexural (antisymmetric) mode resonances do not asymptote properly with increasing order. In

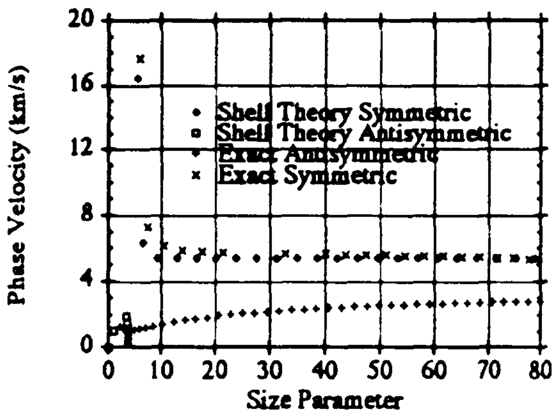

Fig. 1. -- Thick steel shell, membrane model (h / a = 0.1).

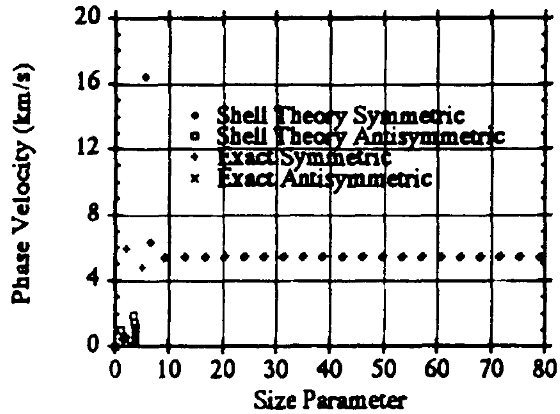

Fig. 2. -- Thin steel shell, membrane model (h / a = 0.01).

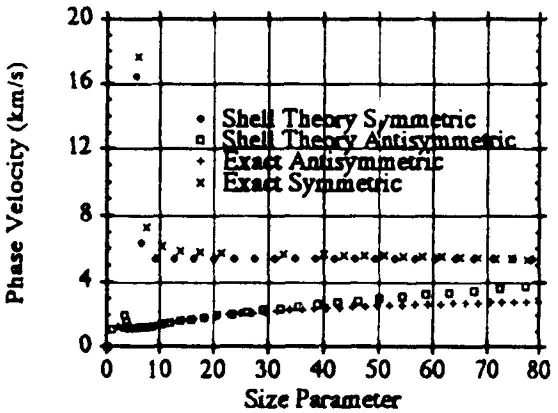

Fig. 3. -- Thick steel shell, vacuum model (h / a = 0.1).

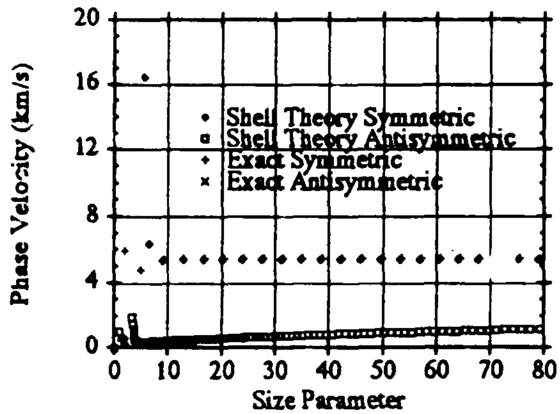

Fig. 4. -- Thin steel shell, vacuum model (h / a = 0.01).

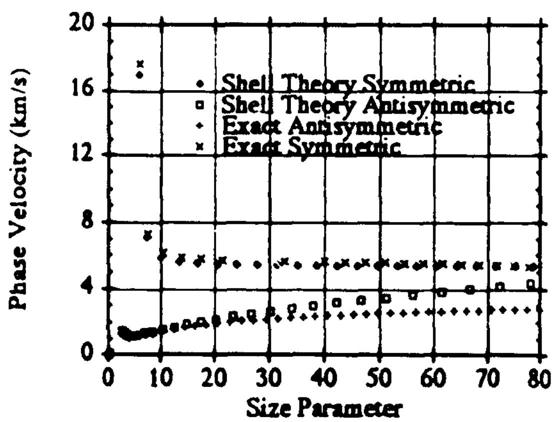

Fig. 5. -- Thick steel shell, fluid loaded (h / a = 0.1).

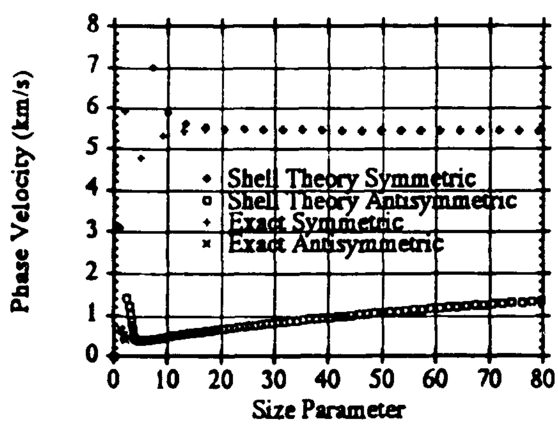

Fig. 6. -- Thin steel shell, fluid loaded (h / a = 0.01).

Fig. 3. we have thick spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by shell theory without fluid loading (vacuum). As in the membrane model the shell theory and exact calculations compare well for the dilatational (symmetric) mode resonances. In contrast with the membrane model, however, the exact and shell theory calculations for this model show much better agreement for the flexural (antisymmetric) mode resonances. This model does not include fluid loading, but does include the effects of rotary inertia. The vacuum shell theory flexural mode resonances do not asymptote for large size parameter *ka* to the exact results, however. In Fig. 4. we see thin spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by shell theory without fluid loading (vacuum). As in the membrane model the shell theory and exact calculations compare well for the dilatational (symmetric) mode resonances except for the first couple of resonances. This vacuum model does not have fluid loading, and has insufficient damping for the first two dilatational (symmetric) mode resonances. Again, the flexural (symmetric) mode resonances show roughly the correct behavior, but it is not possible to tell what the asymptotic value of the phase velocity would be for large size parameter on this scale. Next in Fig. 5. we have a plot of thick spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by shell theory with fluid loading. As in the vacuum case as well as for the membrane model, the dilatational (symmetric) mode resonances compare well for exact and shell theory methods. The flexural (antisymmetric) mode resonances, as calculated by shell theory with fluid loading, do not have the correct asymptotic limit for large size parameter, although they do exhibit the correct behavior for lower values of *ka*. Finally, in Fig. 6. we see thin spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by shell theory with fluid loading. The exact and shell theory calculations agree well for the dilatational (symmetric) resonances and exhibit a marked improvement for the first several shell theory symmetric mode resonances. This is due to the inclusion of fluid loading in the model. The flexural (antisymmetric) mode resonances show the appropriate behavior on this rather limited size parameter scale.

The proper correction for the large *ka* asymptotic resonance behavior is most easily found by including the shear stress distortion along with a Timoshenko-Mindlin[2,3] shape factor. Inclusion of the shear distortion in the potential energy would make the flexural modes asymptote to the coincidence velocity and the shape factor can be adjusted so that they asymptote to the Rayleigh velocity as expected.

4. ACKNOWLEDGMENTS

We wish to thank the Office of Naval Research, the Office of Naval Technology, and NRL-SSC management including Drs. Chin-Bing, Franchi, and Moseley. Dr. Dean is at NRL on an ONT Fellowship. This work was funded by NRL-SSC Program Element 61153N, G. Car, Program Manager.

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